11/12 Recitation

1. How many (nonreal) complex eigenvalues can a 9×9 matrix have?

2. Suppose a 2×2 matrix A has eigenvalue $\lambda = 2 - i$.
   (a) Is A invertible?

   (b) What is the determinant of A?

   (c) Is A symmetric?

3. Recall that for any matrix A, $A^T A$ is positive semi-definite. Is AA^T also positive semi-definite? If so, explain why. If not, provide a counterexample.

4. Let $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

   (a) Orthogonally diagonalize S by hand.

   (b) Is S positive definite?

   (c) Redo part (b) without looking at the eigenvalues of S.
   [Recall that S is positive definite if and only if $\mathbf{x}^T S\mathbf{x} > 0$.]

5. Write a Julia function that takes in an $n \times n$ symmetric matrix and computes $\mathbf{e}_i^T S\mathbf{e}_i$ for all $1 \leq i \leq n$. The code should then either output "S is not positive definite," "S is not positive semi-definite", or "S may or may not be positive semi-definite." How can this code be made more efficient?